Example #1: One-Way Independent Groups Design

An example based on a study by Forster, Liberman and Friedman (2004) from the *Journal of Personality and Social Psychology* illustrates the SAS/IML program for a one-way independent groups design. These authors recruited 42 undergraduate students for a study on "perception and concentration" (p.181). Students were divided into three groups of equal size; the groups were given different thought exercises to perform before completing a set of 12 puzzles; students were asked to imagine solving a series of puzzles in the distant future, near future, or they were in a control group where they solved puzzles without any introduction. The dependent variable was the number of puzzles that each student correctly completed (out of 12).

We simulated a set of data with means and variances similar to those reported by Forster et al. (2004), but exhibiting significant departure from a normal distribution (the degree of departure from normality of the original data set is not known). Following is a sample set of WJGLM code that might be used to analyze these data. Explanation of the code is presented following this listing. It should be noted that it is not necessary to run all of the code at one time, although this is the approach that has been used to generate the results. For example running the code from $Y = \{...$ to the end of the first RUN WJGLM; will produce only the ADF solution using least squares estimators.

```
y = {7, 5, 8, 9, 8, 8, 7, 7, 6, 2, 9, 7, 8, 7, 1, 0, 12, 5, 12, 5, 6, 8, 7, 5, 5, 5, 1, 12, 2, 5, 5, 6, 5, 5, 8, 7, 8, 6, 5, 7, 7, 5};
nx={14 14 14};

c={1 -1 0, 1 0 -1};
opt1=0;
opt2=0;
opt3=0;
print 'Test For Overall Group Effect - ADF Solution';
run wiglm;
```

```
opt1=1;
opt2=0;
opt3=0;
print 'Test For Overall Group Effect - ADF Solution With Default 20% Trimming';
run wjglm;
opt1=1;
opt2=1;
seed=403899;
opt3=0;
print 'Test For Overall Group Effect - ADF Solution With Trimming & Bootstrap';
run wjglm;
c=\{1-10\};
print 'contrast J1 vs J2 - ADF Solution';
opt1=0;
opt2=0;
opt3=0;
run wiglm;
print 'Contrast J1 Vs J2 - ADF Solution With Trimming';
opt1=1;
opt2=0;
opt3=0;
run wjglm;
c=\{1 \ 0 \ -1\};
print 'Contrast J1 Vs J3 - ADF Solution';
opt1=0;
opt2=0:
opt3=0;
run wjglm;
print 'Contrast J1 Vs J3 - ADF Solution With Trimming';
opt1=1;
opt2=0;
opt3=0;
run wiglm;
c=\{0\ 1\ -1\};
print 'Contrast J2 Vs J3 - ADF Solution';
opt1=0;
opt2=0;
opt3=0;
run wjglm;
```

```
print 'Contrast J2 Vs J3 - ADF Solution With Trimming';
opt1=1;
opt2=0;
opt3=0;
run wjglm;
c=\{1-10, 10-1, 01-1\};
print 'Pairwise Contrasts - ADF Solution With Trimming/Bootstrapping';
opt1=1;
opt2=1;
opt3=0;
run bootcom;
c = \{1 - 1 \ 0\};
print 'Effect Size Estimate For J1 Vs J2 - ADF Solution With Trimming';
opt1=1;
opt2=1;
opt3=1;
run wjglm;
c = \{1 \ 0 \ -1\};
print 'Effect Size Estimate For J1 Vs J3 - ADF Solution With Trimming';
opt1=1;
opt2=0;
opt3=1;
run wiglm;
c={0 1 -1}:
print 'Effect Size Estimate For J2 Vs J3 - ADF Solution With Trimming';
opt1=1;
opt2=0;
opt3=1;
loc1=0;
loc2=0:
run wiglm;
quit;
```

The first two SAS/IML statements define the dataset and the number of observations in each group:

```
y = \{7, 5, 8, 9, 8, 8, 7, 7, 6, 2, 9, 7, 8, 7, 1, 0, 12, 5, 12, 5, 6, 8, 7, 5, 5, 5, 1, 12, 2, 5, 5, 6, 5, 5, 8, 7, 8, 6, 5, 7, 7, 5\};

nx = \{14, 14, 14\};
```

In the first statement, the commas used to separate the individual data points delineate the rows of \mathbf{Y} , so that \mathbf{Y} is a column vector with 42 elements. The order of entry for \mathbf{Y} and $\mathbf{N}\mathbf{X}$ corresponds; the first \mathbf{n}_1 rows of \mathbf{Y} correspond to the first element of $\mathbf{N}\mathbf{X}$, the next \mathbf{n}_2 rows of \mathbf{Y} correspond to the second element of $\mathbf{N}\mathbf{X}$, and so on.

The **C** matrix to test the omnibus effect defines a set of linearly independent contrasts among the groups:

```
c=\{1-10, 10-1\};
```

To produce the results for the omnibus test using the ADF solution with leastsquares estimators, the following statements are also needed:

```
print 'Test For Overall Group Effect- ADF Solution';
opt1=0;
opt2=0;
opt3=0;
run wjglm;
```

To produce the omnibus test using the ADF solution and the default option of 20% symmetric trimming, the **C** matrix remains the same, but the following statements are used in place of the statements defined above:

```
print 'Test For Overall Group Effect – ADF Solution With Trimming'; opt1=1; opt2=0; opt3=0; run wjglm;
```

If one only wanted the ADF solution and the default option of 20% symmetric trimming the code would be

```
y = {7, 5, 8, 9, 8, 8, 7, 7, 6, 2, 9, 7, 8, 7, 1, 0, 12, 5, 12, 5, 6, 8, 7, 5, 5, 5, 1, 12, 2, 5, 5, 6, 5, 5, 8, 7, 8, 6, 5, 7, 7, 5};
nx={14 14 14};
```

```
c={1 -1 0, 1 0 -1};
opt1=1;
opt2=0;
opt3=0;
print 'Test For Overall Group Effect - ADF Solution With Default 20% Trimming';
run wiglm;
```

To produce the omnibus test using the ADF solution with 20% trimming and bootstrapping, **C** remains the same, but the following statements are used in place of the statements defined above:

```
print 'Test For Overall Group Effect – ADF Solution With Trimming And Bootstrapping'; opt1=1; opt2=1; seed=403899; opt3=0; run wjglm;
```

To ensure reproducibility of results, we have specified a value for the starting seed for the bootstrap. Once one starting value has been assigned for a bootstrap procedure, it is not necessary to specify another value for any subsequent calls that appear before the QUIT statement. The default value for **NUMSIM_B** (i.e., B = 999) was adopted.

If an omnibus hypothesis is rejected, researchers are often interested in examining focused comparisons among the means, particularly pairwise comparisons. To perform all possible pairwise comparisons using the ADF solution, the **RUN WJGLM** statement is issued J(J-1)/2 times, and **C** is redefined before each invocation. The program code to produce results for the ADF solution when the usual least-squares means are adopted is:

```
c={1 -1 0};
print 'Contrast J1 Vs J2 - ADF Solution';
```

```
opt1=0;
opt2=0;
opt3=0;
run wjglm;
c=\{1 \ 0 \ -1\};
print 'Contrast J1 Vs J3 - ADF Solution';
opt1=0;
opt2=0;
opt3=0;
run wjglm;
c=\{0\ 1\ -1\};
print 'Contrast J2 Vs J3 - ADF Solution';
opt1=0;
opt2=0;
opt3=0;
run wjglm;
```

If one adopts the ADF solution with robust estimators and the default 20% symmetric trimming, the following program code is used:

```
c = \{1 - 1 \ 0\};
print 'Contrast J1 Vs J2 - ADF Solution With Trimming';
opt1=1;
opt2=0;
opt3=0;
run wjglm;
c = \{1 \ 0 \ -1\};
print 'Contrast J1 Vs J3 - ADF Solution With Trimming';
opt1=1;
opt2=0;
opt3=0;
run wjglm;
c=\{0\ 1\ -1\};
print 'Contrast J2 Vs J3 - ADF Solution With Trimming';
opt1=1;
opt2=0;
opt3=0;
run wjglm;
```

To produce pairwise comparisons when the ADF solution with trimming is used in combination with the bootstrap, the **BOOTCOM** module is invoked in order to control the FWER for the set of contrasts. This module is invoked only a single time because **C** defines the entire set of contrasts for which FWER control is required. The following program code is used:

```
c={1 -1 0, 1 0 -1, 0 1 -1};
print 'Pairwise Contrasts - ADF Solution With Trimming & Bootstrapping';
opt1=1;
opt2=1;
opt3=0;
run bootcom;
```

The default values of **NUMSIM_BC** (i.e., B = 699) and **ALPHA** were selected in this invocation of the program.

To compute ES estimates and empirical 95% CIs based on trimmed means and Winsorized variances for each possible pairwise comparison, the following program code is used (assuming the default scaling factor of .642 is adopted):

```
c = \{1 - 1 \ 0\};
print 'Effect Size Estimate For J1 Vs J2 - ADF Solution With Trimming':
opt1=1;
opt2=0;
opt3=1;
run wiglm;
c = \{1 \ 0 \ -1\};
print 'Effect Size Estimate For J1 Vs J3 - ADF Solution With Trimming';
opt1=1;
opt2=0;
opt3=1;
run wjglm;
c=\{0\ 1\ -1\};
print 'Effect Size Estimate For J2 Vs J3 - ADF Solution With Trimming';
opt1=1;
opt2=0:
```

```
opt3=1;
loc1=0;
loc2=0;
run wjglm;
```

For the first two ES estimates, by default the standard deviation for group 1 is used as the standardizer. For the third estimate, for demonstration purposes we have specified the standardizer to be the square root of the average of the variances for the two groups in the contrast.

If it was of interest to control the FWER for the set of ES estimates, then the program user could specify a Bonferroni-corrected value for ALPHA. For example, setting α = 05/3 (i.e., a 98.3% CI) for each ES, the program code would be:

```
c = \{1 - 1 \ 0\};
print 'Effect Size Estimate For J1 Vs J2 - ADF Solution With Trimming';
opt1=1;
opt2=0:
opt3=1;
alpha = .01667;
run wiglm;
c = \{1 \ 0 \ -1\}:
print 'Effect Size Estimate For J1 Vs J3 - ADF Solution With Trimming';
opt1=1;
opt2=0;
opt3=1:
alpha = .01667;
run wiglm;
c=\{0\ 1\ -1\};
print 'Effect Size Estimate For J2 Vs J3 - ADF Solution With Trimming';
opt1=1;
opt2=0;
opt3=1;
alpha = .01667;
loc1=0;
loc2=0;
run wiglm;
```

Furthermore, if the program user elected <u>not</u> to adopt a scaling factor when computing an effect size estimate with robust estimators, then an additional line of code would be inserted in each call to the program. For example, for the first ES estimate (assuming a 98.3% CI), the code would be:

```
c = {1 -1 0};
print 'Effect Size Estimate For J1 Vs J2 - ADF Solution With Trimming';
opt1=1;
opt2=0;
opt3=1;
scale=0;
alpha=.01667;
run wiglm;
```

Finally, we note that the program is not limited to computing an estimate of the ES and confidence intervals for a pair of means. An ES estimate could be computed for the contrast of the average of the first two means and the third mean. In this case, one would specify the contrast matrix as:

$$c = \{0.5 \ 0.5 \ -1\};$$

It should be noted that although the syntax,

$$c = \{1 \ 1 \ -2\};$$

would work equally well for a hypothesis test, if the goal is to estimate the ES for the average of the first two means and the third mean, the syntax

$$c = \{0.5 \ 0.5 \ -1\};$$

is required.

Means and standard errors (SEs) for least-squares and robust estimators for this one-way design are reported in Table 1. The omnibus test statistic based on least-squares estimators is $T_{w,l}/c = 1.80$, with 2 and 24.16 df (p = .1875). With 20%

symmetric trimming the corresponding results are $T_{WJt}/c = 4.98$ with 2 and 16.11 df (p = .0208). When a bootstrap critical value is used to assess statistical significance of the omnibus test, p = 0.0210.

Table 1: Means and SEs for least-squares and trimmed estimators (20% symmetric trimming) for a one-way independent groups design

	Least-squares	Least-squares	Trimmed	Winsorized SE
	mean	SE	mean	
Distant-future (J1)	7.00	0.48	7.30	0.30
Near-future (J2)	6.00	1.06	5.90	1.46
Control (J3)	5.78	0.42	5.80	0.36

For the ADF solution with the usual least-squares means, the program produces the following results for the three pairwise comparisons: groups 1 vs. 2 (distant future/near future): $T_{wJ}/c = 0.74$ with 1 and 18.14 df (p = .4009); groups 1 vs. 3 (distant future/control): $T_{wJ}/c = 3.61$ with 1 and 25.57 df (p = .0688); groups 2 vs. 3 (near future/control): $T_{wJ}/c = 0.04$ with 1 and 17.02 df (p = .8531). For the ADF solution with robust estimators, the program gives the following results: distant future/near future: $T_{wJt}/c = 0.88$ with 1 and 9.78 df (p = .3717); distant future/control: $T_{wJt}/c = 10.09$ with 1 and 17.51 df (p = .0054); distant future/ control: $T_{wJt}/c = 0.004$ with 1 and 10.09 df (p = .9484).

Keselman, Cribbie and Holland (2004) present a number of methods to control the FWER, the probability of committing at least one Type I error for multiple significance tests, when a theoretical critical value is adopted. In this case, we apply Fisher's two-stage procedure which provides exact FWER control because there are only three levels of the grouping variable. Since the stage one omnibus test was rejected (based

on robust estimators), each of the individual pairwise tests is assessed at α = .05. Accordingly, only the second comparison can be declared statistically significant.

The **BOOTCOM** program gives as output the same T_{WJt}/c and df results as the ADF solution with trimming, but specifies an empirical critical value to evaluate the pairwise comparisons. For this example, with the seed specified above, the critical value is 6.11, which means that only the second pairwise comparison (for groups 1 and 3) is judged to be statistically significant.

Finally, the robust ES estimates and their 95% CIs are as follows (using a scaling factor): for groups 1 vs. 2 (distant future/near future) the estimate is 1.12 (-0.78, 3.74); for groups 1 vs. 3 (distant future/ control) the estimate is 1.20 (0.05, 2.84); for group 2 vs. 3 (near future/control) the estimate is 0.03 (-0.80, 0.81). A 98.3% CI (i.e., controlling the FWER to α = .05 for the set of ES estimates) for the first ES is (-1.63, 4.82) if a scaling factor is used. If a scaling factor is not used, the estimate and 98.3% CI for the first ES is 1.75 (-2.13, 7.64).

Reference

Forster, J., Liberman, N., & Friedman, R.S. (2004). Temporal construal effects on abstract and concrete thinking: consequences for insight and creative cognition. *Journal of Personality and Social Psychology*, 87, 2, 177-189.